STONE CIRCLE GEOMETRIES: AN INFORMATION THEORY APPROACH

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Abstract. This article discusses the techniques of A. Thom in deriving geometric designs to fit stone circles and from this background argues for an alternative definition of an hypothesis in scientific research. The definition that is advocated herein is a union of Solomonoff's application of Information Theory to inductive inference, Wallace's Information measures and Halstead's software science measures. This approach is applied to the comparison of Thom's hypothesis against the authors' hypothesis that stone circles are meant to be roughly circular and locally smooth to the eye. The authors' hypothesis is modelled by a fourier series wrapped around a circle. The results from 65 Irish sites show that the authors' hypothesis is favoured at odds of better than 780:1 compared to Thom's hypothesis.

INTRODUCTION

The stone circles of Britain have undergone detailed study and statistical analysis over the past decade. The progenitor of this work, Professor A. Thom, claims that these monuments are set out to accurate geometric designs with the use of a standard unit of length, the 'megalithic yard' (MY) equal to .829m or 2.72 ft (Thom 1967). This claim has been investigated in two recent statistical analyses which both concluded that only the stone circles from Scotland lent some support to Thom's theories (Kendall 1974; Freeman 1976). Both these studies as well as Thom's used about 200 circles whose diameters were estimated by Thom. The statistical examination of a population of diameters for evidence of quantisation is very difficult, as the diameters are not basic data, but are inferred, with uncertain error and bias, from the surveyed positions of individual stones. It is therefore not surprising the previous analyses have produced very tentative conclusions.

1 WHY MEGALITHIC SCIENCE NEEDS A NEW APPROACH

In Thom's studies there are two underlying assumptions. The first assumption is that the geometric design that he has devised for each site is "correct" in some sense. Thom indicates in his writings that he tests a number of designs on a site plan from which a solution is accepted as "the best". Of course, in this context "the best" is poorly defined. Indeed, Angell (1976, 1977) has been critical of this deficient approach as there are no firm criteria by which to select the most appropriate solution. Kendall (1974) felt that analysing the circle diameters is a safe task stating that at least "a circle is a circle". However, he felt that the dimensions of non-circle sites were open to debate of design choice or unconscious bias. Freeman (1976), who also analysed Thom's data, voices stronger misgivings in saying "I disagree, of course, with Thom's claim that we know the design that was originally used".

The second assumption is the selection of a single site dimension to use in an analysis for an underlying quantum. There has been no discomfort expressed publicly by any commentators in using the diameters of circles as an appropriate dimension for analysis, apart from Angell (1976). However Kendall, Freeman and Angell have declared misgivings at using Thom's dimensions from his non-circle designs.

The analyses of both Kendall and Freeman were constrained to work with the data supplied by Thom, that is the diameters of some 200 sites. Thom himself no doubt feels that as he has justified the primary assumption, that is, the choice of geometric design applicable to a site, then no more statistical validation of shape selection is either necessary or appropriate. Thus he embraces the statistical results yielded by the Broadbent tests without equivocation (Thom, 1967), which of course support the existence of the megalithic yard.

The approach in this paper differs from previous analyses, as a fully specified Thomsian hypothesis with specified quantum is compared against our own version of a smooth non-quantal hypothesis. Kendall's analysis took the quantum as unspecified, and Freeman's analysis was designed to estimate the value of the quantum on the assumption that a quantum was used.

It is necessary to construct formally the details of Thom's hypothesis and so eradicate any unjustifiable assumptions, or at least make them the same as the assumptions underlying the hypothesis against which Thom's will be later compared. In attempting to compare Thom's hypothesis (H_T) against the authors' alternative (H_P) some details of H_T had to be inferred from Thom's published analyses, as Thom himself has not to our knowledge yet published a statement of his hypothesis sufficiently precise for testing. We believe our formalisation of H_T represents a fair summary of the state of Thom's work on stone circle geometry up to the present, but concede that it includes some assumptions not inferred from Thom's analyses. These assumptions concern matters not inconsistent with his work. An example is our assumption that the measured radial distances of stones from the centre of a circular site have a Normal distribution around the nominal radius.

The first and perhaps most obvious element of Thom's hypothesis is the definition of all the geometric designs Thom considers were used by the stone circle builders. The second element is the number of different shapes available in the complete retinue of shapes possible. Thirdly, Thom's solutions provide us with the frequency of use of the different designs within any region. These last two elements are very much a function of the stone circles surveyed to date, that is the evidence that has been collected so far. The importance of this fact will be demonstrated further on.

The analysis by Thom of each surveyed site has led him to formulate specific properties of stone circle designs. Firstly, that there was in use a standard unit of length equal to .829m. Secondly, that multiples and sub-multiples of this unit were used in a variety of ways. Thirdly, that there were special dimensional relationships in the design of ellipses, flattened circles and eggs, for example, the use of the pythagorean triple relationship for fundamental design elements.

In an analysis of Thom's theories one would like to eliminate all the underlying assumptions or explicitly incorporate them into the structure of an examinable hypothesis. However, it is also necessary to overcome the objections of Angell (1976) that site dimensions cannot be assessed accurately. Thom's plea that his work *in toto* must be evaluated needs to be satisfied (see discussion in Freeman 1976), though he has never done this himself. Satisfying these pleas means incorporating assessments of not only his quantum, but the perimeter conditions, the different geometric shapes, the use of pythagorean triads and the inherent inaccuracies in the data.

A statistical technique has been devised to fulfil all these requirements. However, there are a number of important facets of this technique that need to be emphasised before its implementation is described. Firstly, this technique can only compare hypotheses. Any number of hypotheses can be ordered in terms of their efficiency at describing a given set of data or evidence, but it does not and cannot prove an hypothesis. Secondly, the comparison is solely based on the evidence available, which in this case is principally the positions of stones at each site. However, further evidence such as the inherent geographical and/or archaeological distribution of sites could be incorporated as evidence if one so desired. Thus the results from this technique are wholly dependent on the evidence available and should new evidence come to light the preference of one hypothesis over another may be reversed.

2 THE COMPETING HYPOTHESES

The archaeological background suggests that the megalithic traditions were started by essentially isolated egalitarian farming communities (Burgess 1974, Burl 1976). Over long periods they developed practical engineering skills but each region retained its individual architectural styles often regulated by local building materials. It is plain that the visual element in all the large monuments is the most important architectural feature, exemplified by wide facades to tomb entrances, sometimes lined in quartz, e.g. Newgrange, the high mounds that enclose superbly corbel vaulted tombs and the long avenues of stones and banks that form entrances to Stonehenge and Avebury. In this context we don't believe there was a specific plan view held by the architects but only a ground view and so generally no specific and detailed ground plan of a site was ever formulated.

Our model is based on our belief that the only specific geometric shape intended by the builders was the circle, and that other clearly non-circular shapes arose as informal modifications to a circle intended to accommodate such features as a flattened facade (in the tradition of passage graves), or elongation along an axis of bilateral symmetry as seen in the Cork-Kerry sites (Barber 1972) of Ireland. Our families of curves are not intended to denote any typological sequence but rather the range of shapes that arise when the plan geometry of sites is a matter of expediency and local smoothness.

The set of functions available under H_T is the union of several parameterised families, e.g.

$t_0(\theta,R)$: circles of radius R,

$t_2(\theta,a,b,\beta)$: ellipses with semi-major axis a, semi-minor axis b and orientation β,

$t_3(\theta,R,\beta)$: flattened circle of radius R and orientation β,

etc.,

where the parameters conform to integer and geometric constraints which Thom has described, and where the expected relative frequencies of the different families can be inferred from his analyses of other sites, circles being the most abundant shape.

Similarly, our H_P is of the general form

$$f(\theta) = R + a_2 \sin 2\theta + b_2 \cos 2\theta + a_3 \sin 3\theta + b_3 \cos 3\theta$$

but for convenience we divide the shapes into the following families;

$f_0(\theta,R)$: circles of radius R,

$f_2(\theta,R,a_2,b_2)$: shapes with second-order Fourier terms,

$f_{3S}(\theta,R,r_2,r_3,\beta)$: shapes with second- and third-order terms with bilateral symmetry of orientation β,

$f_3(\theta,R,a_2,b_2,a_3,b_3)$: shapes with second and third order terms,

these families being listed in decreasing order of expected abundance. f_{3S} is a specialisation of f_3 obtained by putting,

$$a_2 = r_2 \sin 2\beta,\ b_2 = r_2 \cos 2\beta,\ a_3 = r_3 \sin 3\beta,\ b_3 = r_3 \cos 3\beta.$$

Examples of these shapes are shown in Figures 1 and 2, where A is used in place of θ to denote the angle in the polar equation.

3 MEGALITHIC STUDIES TO DATE - ENGINEERING DESIGN

The Thoms and their supporters have completed a great many surveys which constitute a very large body of data. However, at times it is unclear what the exact problem is to be solved, what criteria for data collection have been applied in terms of the accuracy of the data, exactly what data is collected and the manner in which the raw field data should be transformed to make it meaningful for the problem solving processes.

Certainly a great deal of ingenuity and imagination has been well used in the development of suitable solutions for the design of megalithic sites. The designs created by the Thoms and Cowan (1970) and the like lack nothing in perceptiveness. Some iterative testing of solutions has apparently been performed though there is no definitive picture of any systematic or exhaustive comparisons. Plainly, the large range of geometric shapes advocated by Thom indicates many hours of experimentation. However, it is on this point that Thoms' engineering methodology and general scientific methodology are manifestly opposite.

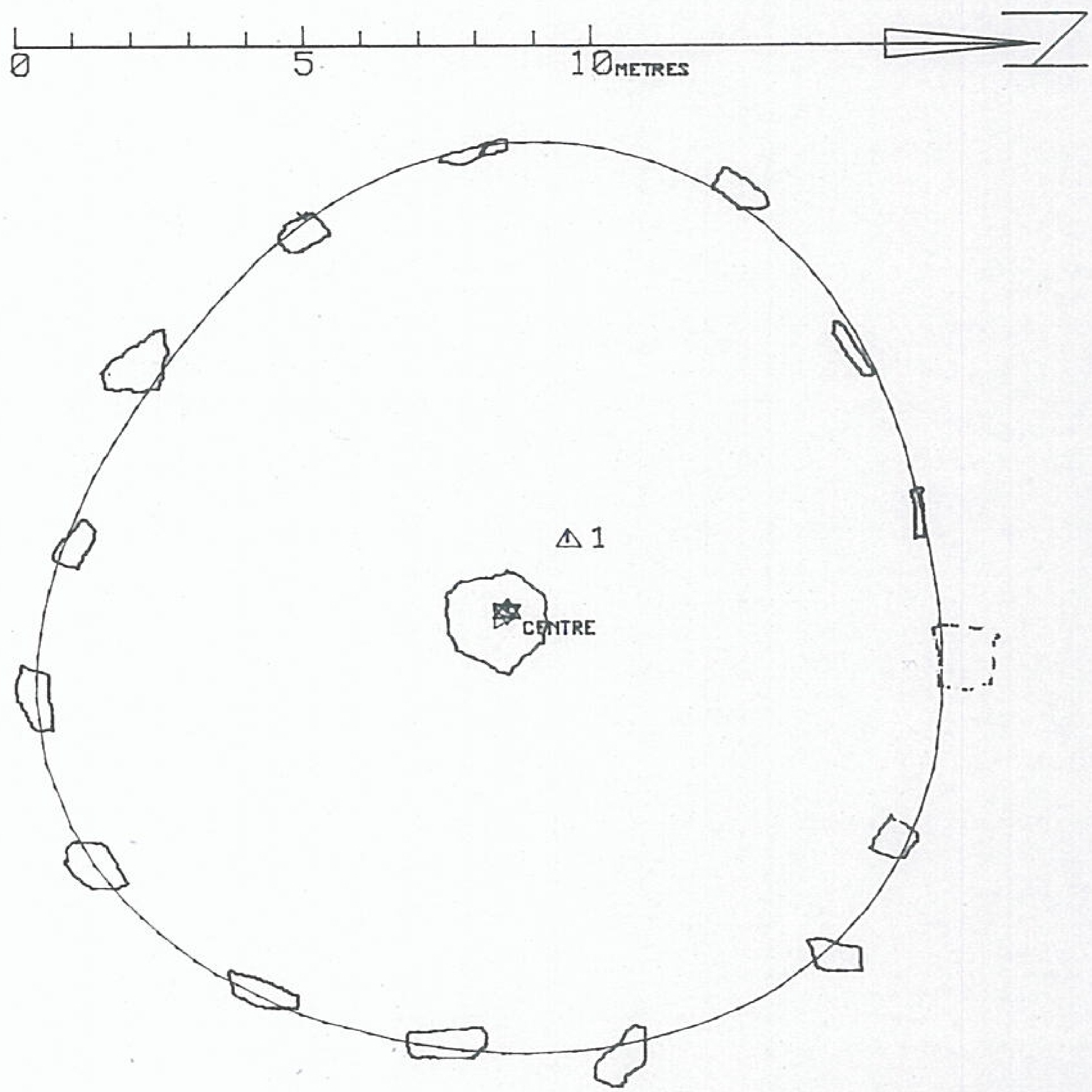


Fig. 1 Fourier Circle – Kenmare

$$F_{3S} = 8.22 + .48 \cos 2(A+.2) - .41 \cos 3(A+.2)$$

The interactive testing of solutions to the problem, i.e. the design of megalithic sites, is regulated by criteria applied subjectively, as is often the case in engineering design. The design solutions for many sites published by the Thoms have been selected on intuitive grounds, on the appeal of the implausibility of alternative solutions or on goodness-of-fit calculations or the circumstantial evidence of comparisons with other sites for which a design solution has previously been published. None of these selection criteria are acceptable under scientific methodology.

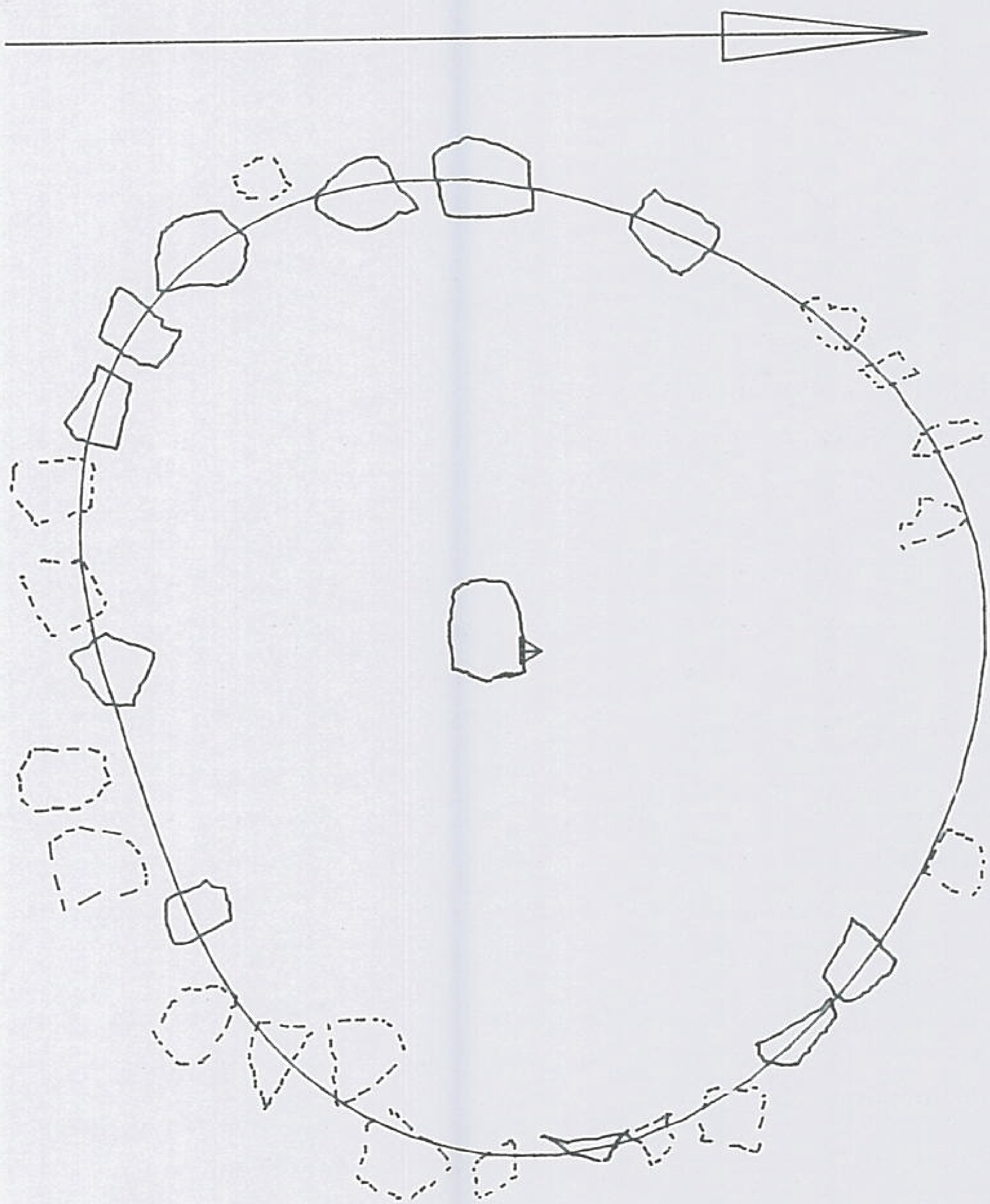


Fig. 2 Fourier Circle – Carrowmore 04

$$F_{3S} = 5.88 - .47 \cos 2(A-1.9) + .32 \cos 3(A-1.9)$$

The application of the scientific method requires the systematic tracing of five steps. The first step is to define the reason for the research and what it is supposed to achieve. This step is quite similar in engineering design. Secondly the modus operandi must be defined in terms of the appropriate research strategies and techniques. In particular it is necessary to decide if one is operating on a deductive strategy, that is to collect data for comparison with an existing hypothesis or an inductive strategy, that is to formulate a new hypothesis.

The third step is to direct the inquiry so that relevant and sufficient evidence is collected for the analysis. Fourthly, the analysis is performed and the outcome is clearly stated either in the form of a new hypothesis (induction) or the extent of the evidentiary support (deduction). The final step and the most important stage for anyone external to the project is the documentation of what was done, what was found and the significance of the findings (Buckley et al.1976).

The current state of the study of megalithic geometry and astronomy is at the inductive inference stage. There has been collected a large number of particular instances and facts and one must move into a tentative generalisation which seems to comprise them all. Many people will complain that this has been done but in fact it has only been done in the engineering sense in that solutions have been supplied within the framework of somewhat arbitrary criteria. The inductive inference has not been completed in the scientific sense as proper testable hypotheses have not been explicitly formulated. As a consequence of this failing the probabilistic analyses of the past have fallen short of shedding much light on the design features of stone circles. For those who consider the inductive stage has been comprehensively fulfilled let them go to any site not previously analysed and apply their deductive reasoning to their expectations of the Thomsian design of that site. They will find very quickly that they have no explicit criteria whatsoever as to the most appropriate astronomical or geometric design for the site.

4 AN HYPOTHESIS - THE CURRENT DEFINITION

For an hypothesis to be good it must fulfil the following criteria (Emory, 1980):

- adequacy, i.e. it must clearly state the condition, size or distribution of some variable or variables in terms of values meaningful to the research task.

- testable, i.e. an hypothesis is untestable if it requires techniques which are unavailable with the present state of the art.
- better than its rivals, i.e. it covers a greater range of facts but is simple in requiring fewer conditions or assumptions.

This last point emphasises the need for an investigator when formulating an hypothesis to find a balance between complexity and simplicity. The Thoms' work falls short on the first of these criteria and its conformity to the third criterion is a major area of debate.

Once a good hypothesis has been formulated and appropriate data collected it is a standard statistical approach to formulate an opposite or null hypothesis. The texts on research methodology emphasise that hypotheses are not proven, but they do say that the statistical tests enable one to accept or reject the original hypothesis. Thus one is provided with a quasi-proof in a manner not unlike the acceptance of an engineering design after the testing and analytical stages.

In this framework of the acceptance or rejection of an hypothesis there is one situation that can cause considerable difficulty. That is the situation where there is no clearcut support for either the hypothesis or its null version. At this point, one can only resort to the dictum of Occam's Razor that "multiplicity ought not to be posited without necessity" (Enc Brittanica).

5 HYPOTHESIS - A NEW DEFINITION

The real requirement of the scientific method is a measure, on a continuous scale, of the extent to which the evidence supports the hypothesis. A continuous measure of evidentiary support would permit the use of a multiplicity of hypotheses and an effective ranking of each. By inference the highest ranking hypothesis would be the most probable explanation of the data and the ranking of hypotheses could quite easily change should new evidence come to hand. The current approach of accepting or rejecting an hypothesis probably extends from our desire to know the 'real' answer, which is not really possible.

This attitude parallels the engineering approach of settling on a specific design or solution in preference to any other.

To create this continuous measure it is necessary to develop axiomatically some new perspectives on the nature of an hypothesis.

AXIOM 1

An hypothesis is a proposition that purports to describe a pattern or order in a set of observable data.

The principle of coding theory is that any message can be coded into another shorter message if there is any pattern in the symbols used in the original message.

Thus a deduction from this axiom is that an hypothesis is an explanation of observable data that purports to describe the data in a message briefer than a full itemisation of the data set itself. Such an explanation is an encoding algorithm and therefore can be assessed of its merit at describing the pattern in the data by the length of the message it generates to describe the data.

AXIOM 2

The aim of the scientific method is to discover the longest pattern sequence in a data set.

This activity is the process of developing more sophisticated hypotheses. If the same pattern or order is found in many data sets and the predictability of that pattern becomes useful for a wide range of deductive analyses then that pattern will become a Law of Nature.

By deduction from the two axioms it can be concluded that an hypothesis that gives a more comprehensive description of the pattern in a data set than its competitors will encode the data into a shorter message than the other hypotheses.

AXIOM 3

The complexity of the statement of an hypothesis is a function of the entities in the hypothesis and their relationships.

An hypothesis statement consists of entities and relationships. However the entities in an algorithm are the operands and the relationships are the operators. As an hypothesis is an algorithm the complexity of an hypothesis' statement is measured as a function of the operators and operands of the algorithm that encodes the data.

The complexity of an hypothesis can therefore be measured as the length of a program coded in some computer or pseudo-computer language. This deduction and the necessity to quantify an hypothesis for the application of Occam's Razor suggests an important corollary.

COROLLARY 1

To compare the complexity of two hypotheses they must be written in a language that does not favour the encoding of either hypothesis by its intrinsic data structures or operators.

The complexity of an hypothesis may be increased to explain data that deviates uncomfortably from the original hypothesis. This expansion creates the appearance of increased evidentiary support for the hypothesis. This is the situation with Thom's work. Alternatively, an hypothesis may be simple and so some of the highly deviant evidence may appear to be inadequately explained.

From the axioms and corollary set down previously and in satisfaction of Occam's Razor it is possible to deduce a suitable measure of evidentiary support.

COROLLARY 2

A continuous measure for the ranking of hypotheses and their evidentiary support is the length of the message that describes an hypothesis statement and the evidence (data) optimally encoded according to that hypothesis.

6 MESSAGE STRUCTURE

In Section 5 it has been established that an hypothesis is a description of a pattern in a data set and therefore can be used to encode the data into a message. It is important that the coding should be optimal in that it should produce the shortest possible message for each hypothesis that is to be compared.

The basis of optimum encoding is Shannon's Information Theory as applied in Huffman coding. Given the probability of each event in a string then Huffman coding enables one to produce the minimum message length for that string. Huffman coding has traditionally been used in computers for binary encoding and thus uses base 2 logarithms and so the length or cost of any message is in units of bits. In the context of this paper an hypothesis is also a predictor of the probability of each event in the message. As the method compares the messages generated by various hypotheses the logarithm base is immaterial and it is more convenient to use natural logarithms. The units are therefore called natural bits or nits. The message is called the "Information Measure" and denoted I_P for hypothesis H_P and I_T for hypothesis H_T (Wallace & Boulton 1968).

The result of the coding activity is the minimum message for each hypothesis and the data. In applying the inverse of Huffman coding the relative probabilities of hypotheses are readily determined from the minimum message lengths.

The total message in any hypothesis consists of two principle sections. They are the fixed overhead of describing the hypothesis and secondly the variable cost of the encoded observational data. Each of these sections may be broken into many components. In the stone circle problem the first section consists of two components:

I1 - is the description of the hypothesis, i.e. a statement of the Fourier family for H_P and for H_T a description of the various geometric designs with their rules of constraint;

I2 - the relative abundances of the various shapes. This component might well be considered as part of the hypothesis but it has been separated to permit a certain flexibility. The relative abundances are considered as parameters of a group of sites and therefore are estimated from the data. Thus it is assumed in both hypotheses that different collections of sites might yield different relative abundances of their various shapes.

The second section of any message is the description of the data, which in this case is the positions of stones encoded according to some geometric design. In this instance, each site message can be divided into three components.

I3 - is the particular geometric shape by which the site is described;

I4 - the description or value of each parameter of the assigned geometric shape;

I5 - the description of each stone with respect to the parameters of the assigned geometric shape.

7 DETERMINATION OF THE COMPLEXITY OF AN HYPOTHESIS-I1

The complexity of an hypothesis as defined in Axiom 3 and Corollary 1 is the I1 component of the Information Measure. This complexity is determined by consideration of a Turing machine.

The Turing machine fulfils the important criterion of not having any intrinsic features that favour any of the competing hypotheses in megalithic studies. Unfortunately to write the necessary programs for Turing machines is a daunting task. However, the study of the structure of programs written in conventional computer languages offers assistance in this matter.

Earlier in this paper it was deduced from an axiom that the complexity of an algorithm is a function of its operators and operands. Halstead (1977) has demonstrated empirically in well-written programs that there is an internal consistency between the number of operators and their frequency.

If η_1 and η_2 are the unique operator and operand counts respectively in a program and N is the total usage of all operators and operands then

$$N = \eta_1 \log_2 \eta_1 + \eta_2 \log_2 \eta_2 \tag{1}$$

Also Halstead derived a function for the volume of a program, i.e. the total length of the program in bits,

$$V = N \log_2 \eta \tag{2}$$

where $\eta = \eta_1 + \eta_2$.

The term $\log_2 \eta$ is the number of bits needed for the unique definition of each operator and operand. The volume V of any algorithm will be a function of the language in which it is written and therefore it is necessary to design a special language for the current needs. If hypotheses are to be compared on an equal footing then the language they are written in should have no intrinsic features that offer a description advantage to any one hypothesis. As well, Solomonoff (1964) says of von Heerden's work that the arbitrary choice of language to describe operators is equivalent to an arbitrary choice of universal Turing machine.

Decoding algorithms for Thom's (H_T) and Patrick's (H_P) hypothesis on the design of stone circles were written in an ALGOL-like language. The language contains the usual arithmetic and logical operators, arrays, the facility for function and procedure definitions and calls, a block structure, FOR loop and CASE statement. The actual decode procedures for uniform and normal distributions are intrinsic to the language. Thus the difference between the two programs is essentially only the difference between the two hypotheses.

Table 1. Halstead's program parameters for H_P and H_T.

	H_P	H_T
η_1	31	40
η_2	71	99
N_1	289	424
N_2	195	326
V(bits)	3229	5339
λ	59.0	63.9

The H_P program decodes for all four families of shapes, i.e. f_0, f_2, f_3, f_{3S} where the H_T program decodes for t_0 (circle), t_2 (ellipse) t_{3A} and t_{3B} (flattened circles). The counts of operators and operands were made for both programs and the results are presented in Table 1. It can be seen that the H_T hypothesis is 2110 bits more than H_P, demonstrating clearly that H_T is by far the more complex hypothesis. The term λ is the language level and is a measure of the sophistication of the language. Typically ALGOL programs yield values between 2 and 4.

The very high values of λ for H_P and H_T indicate that the aim of writing in a language highly suited to the task has been achieved. The fact that λ for H_T is 4.9 higher than H_P shows that the program language is more suited to H_T. If it had been possible to write the programs so that the language levels were identical then the differential in program volume, and therefore hypothesis complexity, between H_P and H_T would be increased even further. It appears that the provision of arrays as a data structure in the programs has assisted the definition of H_T.

8 THE INFORMATION COST OF GEOMETRIC SHAPES - I2 & I3

The abundance of each geometric shape is really a component of the hypothesis overhead i.e. I1. However to permit the flexibility of determining the optimum relative abundances for H_P and H_T it is separated computationally. This relative abundance, i.e. I2, is a multi-state attribute of an entire data set that specifies the frequency of occurrence of each possible shape. For both H_P and H_T there are four permissible shapes. The information cost for a multi-state attribute has been derived in a taxonomic context, by Wallace & Boulton (1968), as the relative abundance of classes.

The message describing each stone circle must be prefixed by the code specifying the particular geometric shape by which the site is described. This piece of code, i.e. I3, is the shape label cost or in the taxonomic sense the cost of specifying the class membership of the site. It has been shown (Wallace & Boulton 1968) that the I2 and I3 components of a message can be combined for computational convenience. The sum of I2 and I3 is derived to be

$$\frac{(T-1)}{2}\ \ln\ (S/12+1) - \ln\ (T-1)! - \sum_{t=1}^{T} (n[t]+1/2)\ \ln\ p[t]$$

where T is the number of states (permissible shapes),

S is the sample size (no. of sites),

n[t] is the number of data items (sites) assigned to state t,

p[t] is estimated by n[t]/S, i.e. the frequency of use of state T.

9 THE OPTIMUM INFORMATION COST OF A SITE - I4 and I5

I5 describes the positions of the stones using a code which would be optimum were the circle indeed set out according to the shape whose family is given in I3 and whose parameters are given in I4. The measured positions of any stone can be specified in polar coordinates as (r_i, θ_i). As neither H_T nor H_P makes any statement about the distribution of θ's, the part of I5 giving the θ's is assumed identical under both hypotheses, and its length is not computed. I5 therefore need only specify for each stone the difference between the measured value r_i and the expected radius $g(\theta_i)$. If these differences are distributed as $N(0,\sigma^2)$ the I5 message length is approximately proportional to $\log \sigma$.

Thus, if a particular geometric shape family has many parameters (e.g. f_3 with 5 parameters), the I4 component for a site assigned to that family will be long, since I4 must specify the value of each parameter. However, one would hope that, by optimum choice of the many parameters, the radial discrepancies could be made small, so reducing the value of σ for the site, and hence reducing I5. On the other hand, if a site is assigned to a simple shape family, I4 will be short, but a poor fit may make I5 longer. Thus there is a trade off between the two components that produce a minimum IM. The full derivations are presented in Patrick (1978) for the case of each geometric shape. However, a simplified model is discussed below.

Consider the program which results if we restrict ourselves to simple circles, the centre of the circle is assumed to be already known and Thom's hypothesis is restricted to integer values for the radius. In this case, the data comprises, for N stones, an ordered set of N values (r_i). An "explanation" of the set of radius data will, under either H_T or H_P, take the form of a message with the following structure:

(σ) (R) $(r_1)(r_2)(r_3)\ldots(r_i)\ldots(r_N)$

Preamble (I4) Body (I5)

The preamble states the standard deviation and mean of the Normal distribution assumed for the radial positions of the stones and forms a simple "hypothesis" about the distribution of the stones. The body gives in turn the value of each radial distance, using a code which would be optimal if indeed the N distances were N independent random values drawn from a $N(R,\sigma^2)$ distribution.

The optimum code for independent random values drawn from a distribution employs a long encoding for improbable values and a short encoding for probable values. The expected length of a message using the optimum encoding for a value of probability p is $(-\log_2 p)$ bits or $(-\ln p)$ nits.

Generally the probability of a stone's position r_i can be expressed as

$$\int_{r_i - \delta/2}^{r_i + \delta/2} \frac{1}{\sqrt{2\pi}\sigma} e^{-(r - R)^2/2\sigma^2} dr$$

where δ is the accuracy of the survey measurements, estimated to be .01 m. Thus applying Shannon's Theory for optimum encoding and summing over N stones the message length for describing all the stones of a site is

$$I5 = -N \ ln\ (\delta/\sqrt{2\pi}\sigma) + \sum_{i=1}^{N} (r_i - R)^2/2\sigma^2 \qquad (1)$$

Now let us say that the radius R can have any value up to a maximum L_R of, say, 35 my for both hypotheses H_P and H_T. Under H_T, the description of R is an integer in the range 1 to 35. Assuming for simplicity that all radii in this range are equally probable a priori, the length of this description is simply (ln35), whatever the value of R.

Under H_P, we can in I4 assign any value to R, but only to a limited precision, since the length of the message must be finite. If we decide to quote R to a precision or least count of U_R, then the I4 description of R has length $\ln(L_R/U_R)$. For instance, if we choose to quote R to the nearest 0.01 my, the description has length ln(35/0.01) or ln(3500), there being 3500 different possible values.

Similarly, under both H_T and H_P, we may assume σ to take values up to some limit L_σ, and if we decide to quote σ to precision U_σ the message length required is $\ln(L_\sigma/U_\sigma)$.
Thus

$$I4 = \ln(L_R L_\sigma) - \ln(U_R U_\sigma) \qquad (2)$$

Now L_R and L_σ are common and identical for both hypotheses and so will make no contribution to discriminating between the hypotheses and shall not be considered further. Note that this is not generally applicable to all parameters for all geometric shapes.

It is necessary to derive expressions for R and σ that minimise the IM given the restraint that the quoted parameters in the message must be one of the set defined by U_R and U_σ. If U_R

is very small then the number of values available to be quoted is very large and so I4 is long and costly. However, a dense set of R values would make it cheap to describe each of the individual stone positions. If U_R is very large then I4 would be cheaply described but the stones may be expensive to encode. Thus as with the definition of the optimum number of parameters there is also an optimum uncertainty for each parameter that minimises the IM.

Whatever the model for the radial distribution of stones, the optimum encoding of their positions on the assumption of a particular intended contour (i.e. perimeter of the geometric shape) has a length which is minus the logarithm of the probability of finding the positions observed, given the assumed intended contour. Thus the estimates for R and σ which minimise I5 solely are the maximum likelihood estimates. However, because R and σ are stated only to limited precision in I4 (a precision of 1 my in the simplified case of R under H_T), the length of I5 using these values will on average exceed the value obtained with maximum likelihood estimates.

Suppose, for a message using H_P, we quote R to precision U_R and σ to precision U_σ. Let (R_0, σ_0) be the values that minimise the I5 component of the message, and let (R_M, σ_M) be the values actually quoted and therefore minimise I4+I5. Define $R_M = R_0 + \Delta_R$, $\sigma_M = \sigma_0 + \Delta_\sigma$, where $|\Delta_R| \leq U_R/2$ and $|\Delta_\sigma| \leq U_\sigma/2$. We assume

$$E(\Delta_R^2) = U_R^2/12, \quad E(\Delta_\sigma^2) = U_\sigma^2/12 \tag{3}$$

Omitting constant terms from (1) and (2)

$$\begin{aligned} I4 + I5 &= N \ln\sigma + \Sigma_i (r_i - R_0)^2/2\sigma^2 - \ln (U_R U_\sigma) \\ &= N \ln\sigma + (v^2 + N(R_0 - m)^2)/2\sigma^2 - \ln (U_R U_\sigma) \end{aligned} \tag{4}$$

where N is the number of stones and

$$m = \frac{1}{N} \sum_{i=1}^{N} r_i \, , \; v^2 = \sum_{i=1}^{N} (r_i - m)^2$$

For any σ, (4) is minimised with respect to R_0 when $R_0 = m$, so we set

$$R_0 = m, \quad R_M = m + \Delta_R \tag{5}$$

giving

$$I4 + I5 = N \ln\sigma + (v^2 + N\Delta_R^2)/2\sigma^2 - \ln (U_R U_\sigma)$$

The precise value of Δ_R will depend on irrelevant details such as the origin chosen for the R_M scale. We therefore substitute the expectation of Δ_R^2:

$$E(I4 + I5) = N \ln\sigma + (v^2 + \frac{N}{12} U_R^2)/2\sigma^2 - \ln (U_R U_\sigma)$$

This is minimised with respect to U_R when

$$U_R^2 = 12\sigma^2/N \tag{6}$$

giving a minimum value

$$E(I4 + I5) = (N-1) \ln \sigma + v^2/2\sigma^2 + 1/2 - \ln U_\sigma - \ln\sqrt{12/N}$$

which is minimised with respect to σ when $\sigma^2 = v^2/(N-1)$.

We therefore set

$$\sigma_0^2 = v^2/(N-1), \quad \sigma_M = \sqrt{v^2/(N-1)} + \Delta_\sigma \tag{7}$$

Substituting its expectation for Δ_σ^2, and minimising with respect to U_σ gives (to second order in U_σ/σ)

$$U_\sigma^2 = 6\sigma^2/(N-1)$$

and

$$E(I4+I5) = -\ln(U_R U_\sigma) + N \ln \sigma_0 + \sum_{i=1}^{N} (r_i - R_0)^2/2\sigma_0^2 + 1$$

The interpretation of the 1/2 nit correction term for each parameter is that the message length is computed using the parameter estimates that minimise only I5. This value must always be at least equal to but usually smaller than the true length of I5 which uses the parameter value actually quoted in a message of minimum length. On average the necessary correction to I5 is expected to be 1/2 nit for each parameter.

Under H_T, the total computable message length is denoted as I_T and there is of course no question of choosing a precision for quoting R, as R can only be an integer in MYs. The length of the preamble for stating R is the logarithm of the number of integer MY values in the assumed range of 35 my. The best integer MY radius is simply the one closest to the mean radial distance, and the best value for σ is found by equating $N\sigma^2$ to the variance of the radial distances about the chosen integer radius. The optimum precision for quoting σ is again close to the expected estimation error.

The computation of the IMs is in practice complicated by several other factors, including the need to estimate the centre of the circle, the possibility of half-integer radii under H_T, allowance for the probable displacement of stones, particularly fallen ones, from their original positions, and of course the existence, under both H_P and H_T, of several different parameterised families of shapes available for the explanation of stone circles. These factors are all incorporated in our analysis and are discussed in more detail in Patrick (1978).

10 DATA GROUPS

The data used to compare the two hypotheses are drawn from four distinct regional groups of sites. The first group is the Cork-Kerry recumbent stone circles of southwest Ireland. Two previous studies of this group have given detailed archaeological assessments and descriptions (O'Nuallain, 1975) and an investigation of the sites' astronomical orientations (Barber, 1973). There are 79 extant sites in this region and 38 have been surveyed by the author (J.P.), but only 35 are suitable for numerical calculations, as the other three are too badly destroyed. The second group of 14 sites are from the Carrowmore passage grave cemetery in western Ireland about 10 km southeast of Sligo. Originally there were over 100 sites spread over a few square kilometres but many of them are now destroyed. These sites do not fit comfortably in the passage grave tradition as no actual passages have been found. However, the circles are formed by contiguous boulders in the manner of a kerb and on the basis of artefactual collections Herity (1974) believes these sites should be assigned to the passage grave tradition. More recent work suggests these sites predated the principal passage grave construction period in the Boyne Valley by over 500 years (Berenhult 1980).

The third group of sites is drawn from the Boyne Valley passage grave cemetery located approximately 45 km north of Dublin. Regrettably only 6 sites from the 16 surveyed sites were in any way useful for analysis. Four of these sites are satellites of the main Knowth tomb. The other sites are the large tomb of Dowth and the smaller mound in the middle of the valley known as 'E'. The fourth group of sites are the Wicklow-Kildare group spread along the western edges of the Wicklow Mts. starting about 20 km southwest of Dublin and running south for 30 km. Nine sites were surveyed and 5 are situated on high hill tops and are almost certainly passage graves. The site on Baltinglass Hill is known from excavations to have been built in at least two phases and so these have been separated bringing the total number of sites to 10. Two sites on the foothills, Athgraney and Boleycarigeen, are stone circles and the last two sites, Broadleas and Castleruddery, have an enigmatic architecture that draws on both the passage grave and stone circle building traditions. Plans of all sites can be found in Patrick (1978).

11 SIMULATIONS

While we have strong reasons to believe that the minimisation of message length provides an absolute test for choice among competing hypotheses, the theory underlying the method is as yet not widely known, nor completely developed. Therefore it was considered desirable to treat ΔI ($=I_P - I_T$) as just another test statistic, and to investigate its sampling distributions under H_P and H_T. This study is not yet complete but Monte-Carlo calculations have provided estimates of sampling distributions of ΔI when H_T and H_P are restricted to circular models, and the true population is either a population of Thom circles, or a population of circles of arbitrary radius.

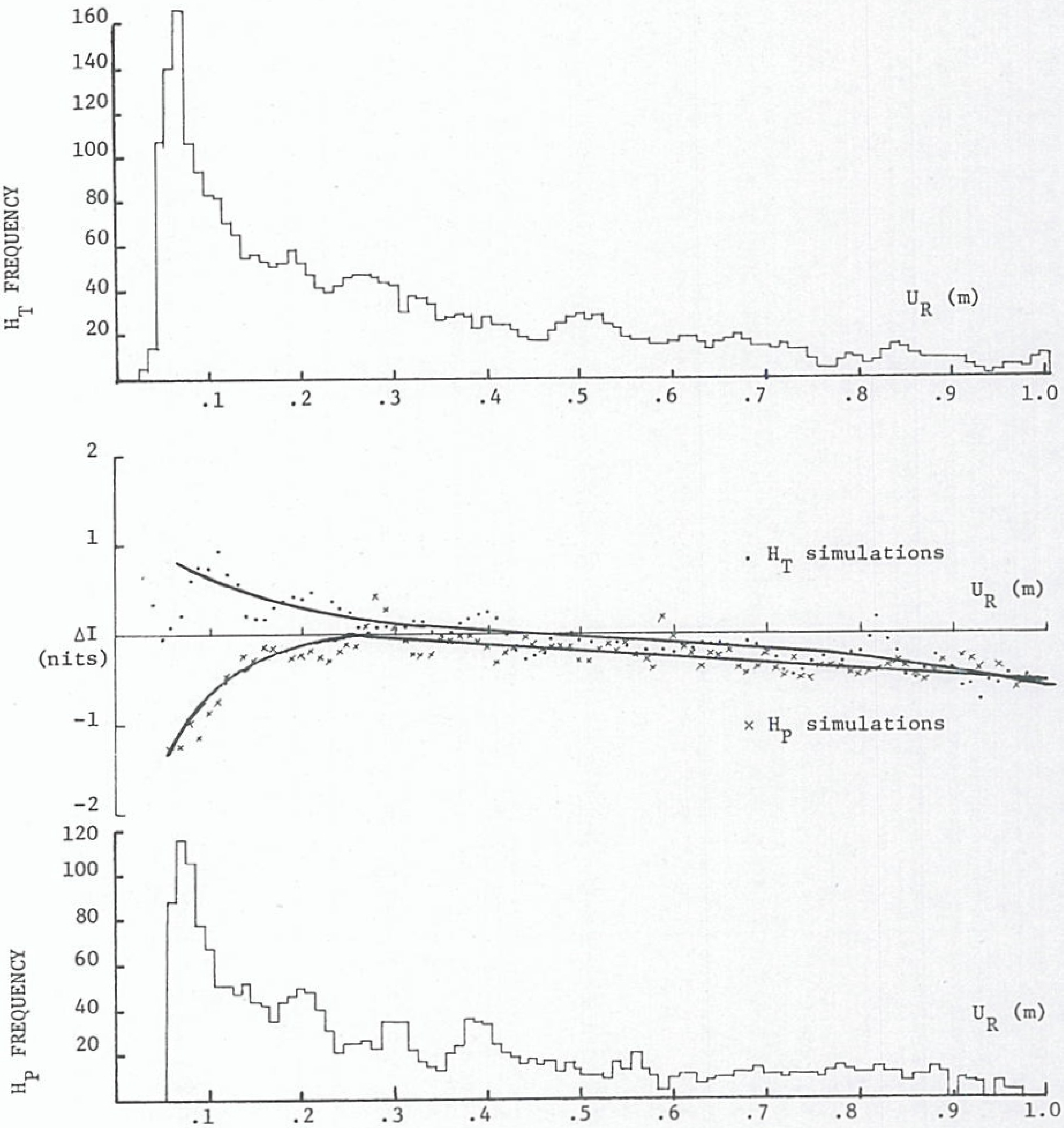


Fig. 3 Cork-Kerry simulations: Combined Groups I and II - Three point moving average graph of ΔI as a function of U_R for H_P and H_T. The histograms show the number of values used to determine each average ΔI.

To ensure that these distributions would be relevant to the field data, the artificial populations were designed to have the same numbers and angular distributions of stones as the real sites, and to have similar ranges of radii. As the real sites fall into two rather distinct groups, two Monte Carlo calculations were made, one with artificial populations resembling the Carrowmore sites, the other mimicking the Cork-Kerry sites, which are rather smaller and have on average only half the number of stones.

Figure 3 shows the results for the Carrowmore sites with 390 and 280 simulations of H_P and H_T circles respectively. This figure is a plot of the mean ΔI as a function of the uncertainty in the radius, U_R. The two curves in Figure 3 indicate that ΔI ceases to distinguish H_P and H_T populations when U_R exceeds about .45 m. For very large U_R, ΔI always favours the H_P hypothesis regardless of whether the population conforms to H_P or H_T, because the quantisation of an H_T population is so swamped by noise that there is no justification for ascribing any particular quantum number to R. One can expect good discrimination between the hypotheses if U_R is smaller than about .4 m but very little or no discrimination for larger values of U_R. There were 720 and 1020 simulations of H_P and H_T respectively for the Cork-Kerry sites, which yielded similar patterns except the limiting value of U_R was about 0.30 m,which was caused by a smaller number of stones per site.

12 RESULTS

The discussions presented in this paper are confined to looking at the overall and regional results. The details from each site will be discussed elsewhere. All IMs were computed using the centroids of the stones. There is some suggestion that more stones fall outwards than inwards and the evidence for this is presented in Patrick (1978).

The implementation of the IM technique involves computing for each site the I4 + I5 elements of the IM for each shape. Table 2 is a list of these values for all the Carrowmore sites. It can be seen readily that for many sites the IMs differ very little for the different shapes. Take for example Carrowmore 19 where the difference between the f_2, f_3 and f_{3S} IMs is only 0.57 nits. Any message that describes a set of sites must communicate the dictionary of shapes available as defined in the hypothesis statements and must describe the shape

Table 2: The (I4 & I5) IM components of the Carrowmore sites for each shape. The a_{3A} and a_{3B} IMs refer to Thomsian Flattened circles but with the unbiased estimate of the radius rather than an integral MY radius.

	H_P				H_T				H_A	
SITE	f_0	f_2	f_3	f_{3s}	t_0	t_2	t_{3A}	t_{3B}	a_{3A}	a_{3B}
01	34.94	32.34	33.32	33.60	35.72	39.33	31.22	32.08	31.51	30.31
04	27.91	22.48	20.83	20.76	28.45	26.20	22.29	19.42	22.77	18.17
07	34.83	32.71	34.62	34.95	34.79	37.05	35.86	32.93	34.96	33.93
09	11.72	10.95	-	12.41	11.90	14.54	13.49	13.33	13.12	13.22
15	15.01	16.41	17.23	17.99	15.15	27.17	16.23	18.35	16.68	18.99
18	22.82	22.86	-	24.80	22.54	26.78	24.52	21.70	24.54	23.21
19	54.76	49.26	49.83	49.51	55.41	50.89	48.91	50.95	47.42	51.24
26	26.76	20.48	21.42	20.65	25.67	23.70	19.84	22.07	20.17	18.49
28	44.06	46.01	45.89	46.15	43.16	51.36	46.05	49.66	46.16	50.49
32	24.80	22.45	23.45	23.57	26.26	28.10	20.07	22.66	20.92	22.82
36	41.67	41.85	43.97	44.18	41.92	46.13	43.21	42.68	43.27	42.05
37B	25.93	25.35	24.37	26.01	26.75	31.50	27.70	27.44	26.80	26.68
48	10.74	11.30	-	13.37	9.84	13.74	12.57	12.71	12.94	12.80
57	34.92	29.08	27.09	26.64	35.60	34.40	27.72	31.68	28.71	30.21
TOTAL	410.87	383.53	-	394.59	413.16	450.89	389.68	397.66	389.97	392.61

allocated to each site. It is necessary to search throughout the table of site IMs for the combination of site IMs and dictionary and shape costs that minimise the total IM. An algorithm for a search technique can be found in Boulton (1975).

Thom has argued that the British data on which he bases his hypothesis should not be subdivided but studied as a total data set (see Patrick and Butler 1974). To apply the IM technique in this manner one must search for the optimum shape combination over the 65 Irish sites. It was evident very early in the analysis that the shapes of f_3 and t_2 (Thomsian ellipse) were of no use to their respective hypotheses. The cumulative totals for each hypothesis for each data group are presented in Table 3. The difference between the two hypotheses is 0.43 nits, which is insignificant. However it is evident that Thom's Type B flattened circle is of no use and so need not be described in the dictionary. The new total for H_T is 908.76 nits which gives H_T a lead of 3.78 nits. This can be interpreted as an odds ratio of 44:1 ($e^{3.78}$:1) in favour of H_T. This represents a marginal advantage for H_T but must be viewed in the presence of the I1 part of the IM, where we believe Thom's hypothesis requires a much lengthier description than does the Patrick hypothesis. A comparison of H_T amd H_P solutions for Carrowmore 26 can be seen in Fig 4.

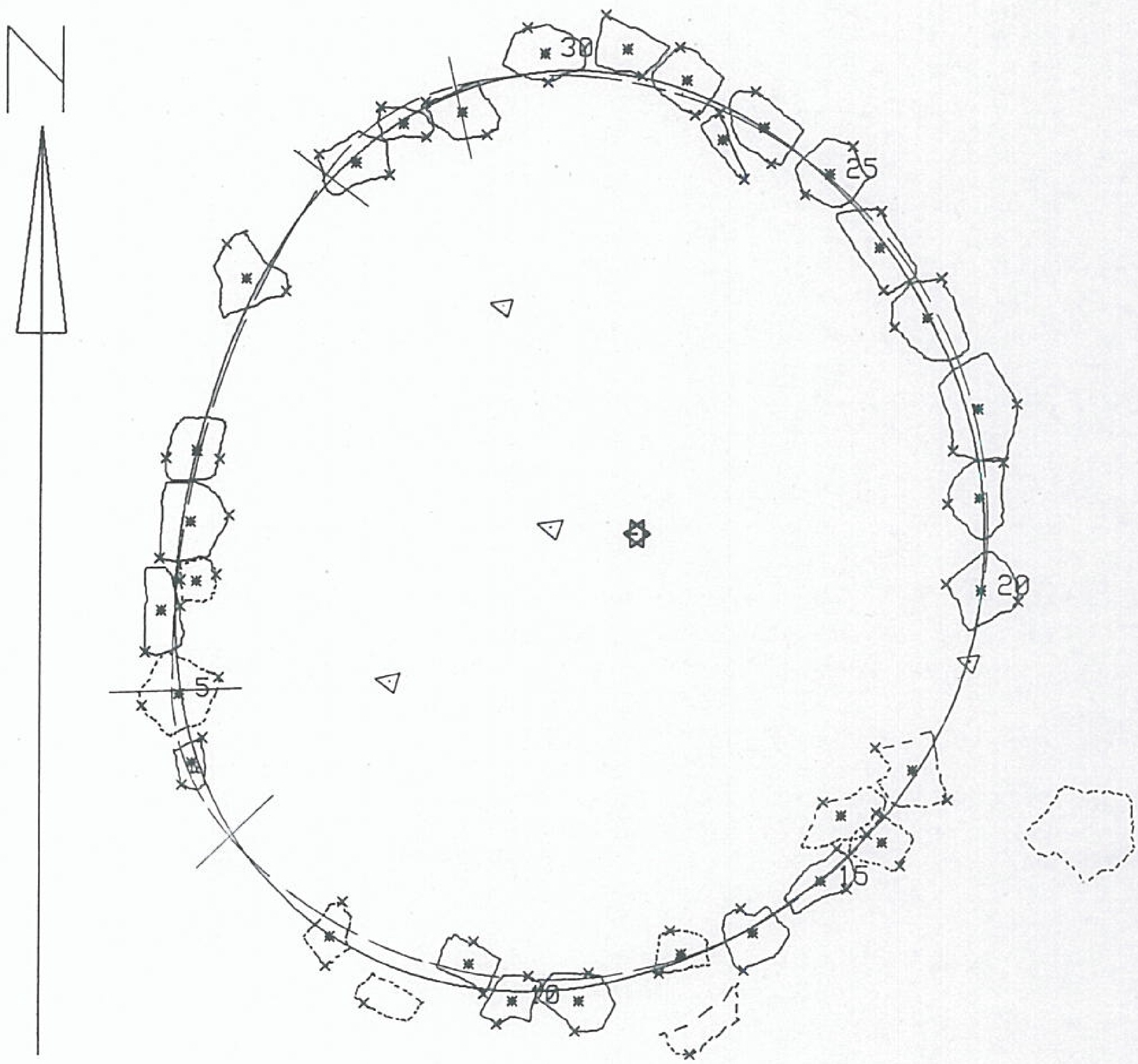
13 THE REGIONAL RESULTS

The overall results presented in the previous section ignore any groupings of the data and merely provide the optimum IM on the assumption that the shape frequencies are independent of any regional groups. This is consistent with Thom's presentation of his own work on English and Scottish data. However we consider that the Irish data has a clearly defined classification of four groups based on geographical distribution and archaeological evidence. There are many features of the sites which differ markedly between different groups, yet are relatively uniform within a given group (Herity, 1974; O'Nuallain, 1975). Were the data necessary to describe these features added to our geometric data, there is little doubt that a message conveying the enlarged data set would be minimised by a classification into groups corresponding closely to the geographic groups. If our results show geometric differences between the groups, this will reinforce the validity of the classification.

Fig. 4 Comparative examples of H_T and H_P solutions for Carrowmore 26.

Broken line f_{3s} : $r_0 = 7.56$, $r_2 = -.43$, $r_3 = 0.22$, $\beta = 16^0$, $\sigma = .25$

Full line a_{3a} : $r = 7.82$, $\beta = 163^0$, $\sigma = 0.27$



A message that is intended to describe the total group of 65 sites making use of their geographic grouping will now have to provide both class dictionaries and class labels that describe the allocation of each site to one of four archaeological groups. The sites are distributed in the proportions of 14:35:10:6 for Carrowmore, Cork-Kerry, Wicklow-Kildare and the Boyne Valley respectively. This has the cost of 80.45 nits but is the same for both H_P and H_T and so cannot contribute to discrimination between the two hypotheses. Now, for each group it is necessary to encode the dictionary of shape families available and the shape that each site belongs to. Table 4 presents a list of the IM for each regional group for a selection of shape

Table 3: The total optimum IMs for each hypothesis over 65 sites. Dictionary and label costs are not included in each regional total but are only determined from the total distribution.

GROUP	H_P	Shape Frequency f_0, f_2, f_{3S}	H_T	Shape Frequency t_0, t_{3A}, t_{3B}
CARROWMORE	378.31	(7,6,1)	376.10	(8,6,0)
CORK-KERRY	276.56	(35,0,0)	270.99	(34,1,0)
WICKLOW	158.21	(3,5,2)	174.73	(5,5,0)
BOYNE VALLEY	48.33	(3,1,2)	51.25	(4,2,0)
	861.41		873.06	
DICTIONARY AND LABEL COSTS	51.13	(48,12,5)	39.05	(51,14,0)
	912.54		912.11	

Table 4: List of accumulated IMs plus shape label costs for each data group for a selection of the optimum shape combinations under H_P and H_T. Note that each table entry includes both a shape dictionary and shape label message cost according to the frequencies shown in brackets. Each group minimum is encircled.

SHAPES	CARROWMORE	CORK-KERRY	WICKLOW KILDARE	BOYNE VALLEY	TOTAL
H_P					
$f_{0,2,3,3S}$	393.80 (5,6,1,2)	295.84 (30,3,0,2)	171.40 (1,5,1,3)	58.23 (2,1,1,2)	919.27
$f_{0,2,3S}$	388.67 (0,12,2)	282.85 (35,0,0)	168.80 (1,6,3)	55.99 (4,0,2)	$\boxed{896.31}$
$f_{0,2}$	389.68 (5,9)	280.04 (35,0)	169.19 (2,8)	58.29 (3,3)	897.20
$f_{0,3S}$	392.20 (8,6)	281.54 (34,1)	173.30 (2,8)	$\boxed{54.45}$ (4,2)	901.49
$f_{2,3S}$	386.57 (12,2)	–	167.48 (7,3)	62.39 (4,2)	–
f_0	410.87	$\boxed{276.56}$	199.38	66.67	953.48
f_2	$\boxed{383.53}$	–	$\boxed{166.68}$	62.42	–
f_{3S}	394.59	–	172.34	–	–
H_T					
$t_{0,2,3A,3B}$	389.00 (6,0,5,3)	288.73 (32,0,2,1)	184.67 (3,3,1,3)	59.72 (3,0,2,1)	922.12
$t_{0,3A,3B}$	387.28 (6,5,3)	280.30 (35,0,0)	184.56 (5,5,0)	57.60 (4,2,0)	909.74
$t_{0,3A}$	$\boxed{386.79}$ (8,6)	277.46 (35,0)	182.70 (5,5)	$\boxed{56.02}$ (3,3)	$\boxed{902.97}$
$t_{0,3B}$	391.90 (6,8)	282.11 (33,2)	193.18 (5,5)	62.29 (3,3)	929.48
t_0	413.16	$\boxed{274.01}$	199.53	65.05	951.75
t_{3A}	390.53	–	$\boxed{181.12}$	62.46	–

combinations for both hypotheses. The number of sites assigned to each shape is shown in brackets. The results show that for both H_P and H_T the use of the complete range of shapes, i.e. using the absolute minimum IM for each site, is one of the most expensive ways of describing the data. The sixth column of totals shows that for H_P the optimum message is a $f_{0,2,3S}$ shape combination and for H_T it is $t_{0,3A}$. The detailed results show that for both H_P and H_T the Cork-Kerry group are best described as only circles with no ovoids at all. The Boyne Valley group has the minimum IMs for the $f_{0,3S}$ shapes under H_P and $t_{0,3A}$ under H_T. The Carrowmore and Wicklow groups yield the minimum IM under H_P as f_2 while under H_T the $t_{0,3A}$ shape combination is the minimum. Thus we can say that for H_P the f_0, f_2 and f_{3S} shapes are all needed to provide the optimum solutions and for H_T the t_0 and t_{3A} shapes only are necessary.

It is evident that most regional groups have substantially different optimal shape frequencies and so the cumulative totals produce a somewhat different result to the overall results presented previously. For H_P the $f_{0,2,3S}$ shape combination yields an IM of 896.31 nits whilst the H_T optimal shape combination is $t_{0,3A}$ with a value of 902.97. The difference represents an odds ratio of 780:1 (i.e. $e^{6.66}$: 1) in favour of H_P.

Under H_T the four groups yield t_0 to t_{3A} ratios of 8:6, 35:0, 5:5 and 3:3. Thus the only significant correlation of geometry with group is the absence of non-circular shapes in the Cork-Kerry group. Under H_P, the f_0, f_2, f_{3S} ratios are 0:12:2, 35:0:0, 1:6:3 and 4:0:2. This shows, besides the circularity of all Cork-Kerry sites, a significant difference between the fourth (Boyne Valley) group and all others. Thus H_P benefits more than H_T by a geographic classification.

It was requested, well before the development of this technique (Patrick 1975), that groups of sites already defined by prior research should be analysed independently. This request was based on the archaeological evidence that prehistoric Ireland and Britain was occupied by small independent tribal units. The different shape frequencies among regional groups revealed by this study are consistent with the spirit of the Patrick and Wallace hypothesis and vindicates the early considerations.

14 CONCLUSIONS

The presented results do not reveal a clear picture of which hypothesis is preferable. On a regional basis H_P is favoured at odds of 780:1. The regional group data indicate that H_P requires the three shapes f_0, f_2 and f_{3S} and that these three shapes have markedly different geographic distributions. This is gratifying as it is consistent with the general description of H_P. However it was unexpected that the f_0 shape would so completely dominate the Cork-Kerry group. In retrospect this is a sensible result as most of the sites have very few stones and so it is not possible to provide evidence to support the more complex designs. This problem introduced the question as to whether one should consider that only one shape family is present; i.e. f_{3S}. Then many sites would merely have zero fourier coefficients. Fourier family, f_{3S}, descriptions for all sites using zero coefficients would be more costly compared to f_0 or f_2 descriptions but would have no shape dictionary or label costs whatsoever. Whilst such a classification might be advantageous for describing substantial numbers of sites where archaeological classifications are ignored it would diminish the visibility of shape to region correlations that can appear, as, for example, in the Cork-Kerry group.

The Carrowmore and Wicklow results are gratifyingly consistent with H_P because for most sites it is not possible to discriminate between the use of f_2 and f_{3s} and to a lesser extent the f_0 shapes. Thus most sites have fairly ill-defined shapes which is consistent with our hypothesis. The comparison of the H_P results to H_T for Carrowmore indicates that Thom's flattened circle designs are quite efficient at describing these sites with their flattened facades. Thom's circle t_0 is an efficient shape though the simulations show there is no distinguishable difference between the two hypotheses for circles set out with the average uncertainty of these sites. The Wicklow sites provide an identical picture for H_P where f_0, f_2 and f_{3s} are highly competitive with each other. However Thom's hypothesis is hopelessly inefficient at describing these sites.

The Cork-Kerry results provide a direct comparison of just the circle shapes without the complexities of shape labels. The slightly shorter message length for H_T suggests there may be some merit in Thom's arguments for a quantum. However, the advantage held by H_T is not as large as one would expect from the simulation results were H_T true.

We have investigated a weak pacing hypothesis, viz. that each circle was set out with a radius equal to an integral number of paces, where the "pace" for each site is a random variable drawn from N(0.8,0.1) (in metres). This hypothesis leads to a prior distribution of radii having modest peaks up to about 5 m, but thereafter virtually uniform. Its use leads to an IM (I4 + I5) for the Cork-Kerry sites smaller than does H_T. Thus, the advantage of H_T over H_P for these sites is evidence only of a very rough quantisation.

If we restrict ourselves to just the circles f_0 and t_0 we can say for both the Carrowmore and Cork-Kerry sites that H_P and H_T generate essentially the same message length. However, H_P describes the sites on the assumption that the radii are uniformly distributed, but to achieve this description an optimal quantal subdivision of the data is determined. The nodal positions of this subdivision are separated at the uncertainty in the radius, U_R, which averages .42m and .28m for eight Carrowmore sites and the Cork-Kerry group respectively. This gives average nodal points at .42 and .28m intervals respectively, which, averaged over the 43 sites, is .30. Now Thom's hypothesis as we infer it from his published results is that 1 my radii, i.e. .829 m occur with a frequency of .7 and $\frac{1}{2}$ my radii, i.e. .415 m, have frequency .3. Thus if one conjures up a single "effective" quantum that would occur with equal frequency it would seem to be a value between .415 and .829 m. The conclusion is that because of the inaccuracies of the data, a description of sites having uniformly distributed radii using Thom's t_0 shapes, is little different to describing the sites as having uniformly distributed radii with an average uncertainty of measurement of about 0.3 m. Thus Thom's quantum hypothesis imitates very closely the random hypothesis in the area of uncertainty that the Irish data falls into.

The evolution of Thom's hypothesis can now be seen in a new perspective. His initial hypothesis claimed the existence of a quantum of 5.44 ft (Thom, 1955). Then the quantum was revised to 2.72 ft (Thom, 1961). This was followed by the addition of the 1.36 ft quantum at a frequency of 30% (Thom, 1967). The last of the inconsistent observations that were not satisfactorily explained were subjected to perimeter conditions (Thom 1967). Thus we can see that as more data became available a deepening complexity evolved to successively pick up the inconsistent observations. There can be little doubt that a "megalithic yard" was never used to set out the Irish sites. Thom's flattened circle geometric design is a sensible interpretation of the stone positions but certainly fits no better and often much worse than the fourier circles. Thom's hypothesis is so complex and involved that it takes many pages of written text to describe. The Patrick and Wallace hypothesis on the other hand can be presented much more briefly and must be favoured a priori. Given this condition H_T must be rejected in favour of H_P for the Irish data analysed herein.

The two previous analyses of Thom's stone circle diameters have considered that only a 1 my quantum was used (Kendall 1974; Freeman 1976). It is possible to use the IM results to remove the effect of the 1/2my quantum. There are 23 sites that use a 1/2my quantum for the optimum solution and likewise 42 sites that use a 1my quantum. These 23 sites must be changed to the nearest 1my quantum which results in adding 67.21 nits to the overall total. However as the probability of 1my quantum is now 1 so there is a total correction to the 65 sites of -42.67 nits. Thus the overall result of removing any 1/2my solutions is to add 24.54 nits to I_T. Whilst this is an over-estimate of the correction, because changes in the optimum shape have not been taken into account, it is evident that the simplified H_T would fare even worse than it does at present against H_P.

The group results demonstrate the importance of considering the sites in their archaeological groupings. The Cork-Kerry stone circles are circles some of which are set accurately and some rather carelessly. The design plans of the passage graves are certainly not circles but are the product of other architectural considerations where the builders created a wide somewhat flattened facade to give an impressive entrance to the tomb. The kerb was completed to present a continuous wall and positioned to look like a neat curve without perturbations that would offend the eye whilst maintaining the general scale of the monument as defined by the size of the front facade region. As each small section of the kerb was kept smooth, large scale variations were imperceptible, resulting in the kerb having fluctuating sharper and flatter curves that are only now made apparent by our modern-day plans.

Professor Thom's geometric designs and megalithic yard are, in our opinion, somewhat extravagent extrapolations of the evidence available. His hypothesis is not competitive for the Irish sites tested herein and we feel this must intimate a similar result for the British sites once they are evaluated by the technique developed here.

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